Mechanisms for the Development of Martensitic Transformation of a Face-Centred Cubic Structure into a Body-Centred Tetragonal Structure

Mechanisms for the Development of Martensitic Transformation of a Face-Centred Cubic Structure into a Body-Centred Tetragonal Structure*

Hideji SUZUKI

The Research Iustitute for Iron, Steel and Other Metals

(Received October 24, 1953)

Synopsis

Two mechanisms of the martensitic transformation of a face-centred cubic structure into a body-centred tetragonal structure were discussed. Some considerations concerning the characteristics of the defects at the interphase boundary were first given. According to these considerations, the transformation takes place depending upon the mechanisms of creation of perfect dislocations, which would release the strain energy caused by the transformation. Two models were proposed concerning the creation of perfect dislocations in a set of planes with regular spacing. One is a static process, in which the length of the dislocation is increased continuously by consuming the strain energy or directly the transformation energy, and the other is a dynamical process, in which the dislocation pairs are created by consuming potential energy such as stacking fault energy or strain energy stored by freezing the kinetic energies of atoms during the transformation by means of rather a complicated process.

The calculated habit planes, lattice relations and deformations associated with the transformation, based on these models and on the assumption of minimum strain energy, seemed to agree with the observations; especially, these two models could sufficiently illustrate the origins of the two different modes of transformation, Umklapp and Schiebung.

I. Introduction

In recent years, many investigations have been made to solve the martensite problem; especially, the atom movements accompanied by the martensitic transformation were discussed on the basis of experiments on the macroscopic deformation associated with the transformation, the lattice relation between the matrix and the product, and the habit plane of the product.(1)~(4) These investigations, however, were concerned only with the atom movements satisfying the above-mentioned three relations, but not with the mechanism of formation or motion of defects, only by means of which the required rearrangements of atoms would take place. The characteristic features of martensitic transformation, however, would depend on the condition of formation and motion of these defects.

Of course there have been a few attempts in which the behaviours of the above-mentioned defects are explicitly taken into the consideration, such as the theory

* The 740th report of the Research Institute for Iron, Steel and Other Metals.

(1) J. S. Bowles and C. S. Barrett, *Progress in Metal Physics*, III (1952), 1.
(2) M. A. Jaswon and J. A. Wheeler, Acta Cryst., **1** (1948), 216.
(3) J. S. Bowles, Acta Cryst, **4** (1951), 162.
(4) E. S. Machlin and M. Cohen, J. Metals, **3** (1951), 1019.

of transformation in pure cobalt by Christian[5] and in iron-base alloys by Frank[6]. The transformation of a face-centred cubic into a hexagonal close-packed structure in cobalt, however, is very simply described in terms of propagation of half dislocations, while the body-centred tetragonal structure could not be realized only through a regular displacement of close-packed atomic planes in a face-centred cubic lattice. The consideration by Christian, therefore, could not be extended directly to the transformation of a face-centred cubic structure into a body-centred tetragonal structure. Moreover, his mechanism of formation of required half dislocations is not regarded as reasonable[7]. In Frank's theory, the mechanism of formation of perfect dislocations is not clear, and he has only suggested that these dislocations would be created by the stress field at the end of the martensite plate. The stress field, however, would not be so strong as to create a loop of dislocation, because the dislocations created already at the interphase boundary will move to release the stress field on account of their screw character. Besides, it is well known that there are two habit planes $\{225\}_A$ and $\{259\}_A$ in the transformation of the face-centred cubic structure into the body-centred structure[8] and that the mechanism of propagation, especially, the rate of propagation is markedly different in these two cases[9][10]. In the theory of martensitic transformation, therefore, it will be the most important problem to answer the question why these two different mechanisms take place. In the present research, some characteristics of defects at the interface between the matrix and the product phases were first discussed and then the models for two mechanisms of the transformation of the face-centred cubic structure into the body-centred structure were considered, and finaly the habit planes, the lattice relatoins and the macroscopic deformations associated with the transformation were derived from these models, using the conditions of formation and motion of dislocations and the assumption of the minimum strain energy.

II. Characteristics of defects at the interphase boundary

When the matrix lattice turns into the product only through parallel displacements of close-packed atomic planes, the defects at the interphase boundary can be described in terms of the usual imperfect dislocations. The transformation of a face-centred cubic structure into a hexagonal close-packed structure belongs to this category*. The transformations of a face-centred cubic structure into a body-centred tetragonal or of a body-centred cubic structure into a hexagonal

(5) J. W. Christian, Proc. Roy. Soc., **A 206** (1951), 51.
(6) F. C. Frank, Acta Met., **1** (1953) 15.
(7) T. R. Anantharaman and J. W. Christian, Phil. Mag., **43** (1952), 1338.
(8) A. B. Greninger and A. R. Troiano, Trans. AIME., **140**(1940), 307.
(9) P. Förster and E. Scheil, Z. f. Metallk., **32** (1940), 165.
(10) H. Suzuki, T. Homma and S. Takeuchi, to be published in this Report. H. Suzuki and T. Homma, J. Metals, **4** (1952), 519.

* Of course this is true only in an approximation. Actually the transformation is also accompanied by changes in the area of close-packed atomic planes and in the distance between them. These changes however, are far smaller than those in other cases.

close-packed structure, however, cannot be the case. The interphase boundary between these structures cannot be described by an array of the usual imperfect dislocations on account of other modes of deformation than the parallel displacement of atomic planes during the transformation. Of course, we can suppose that there will be several arrays of imperfect dislocations and that some of them will climb* during the progress of trasformation. The imperfect dislocations thus supposed, however, depend upon the direction of the interphase boundary as well as upon the orientation relation between the matrix and the product. It is, therefore, rather inconvenient to describe the defects at the interphase boundary in terms of the usual imperfect dislocations.

In general, the product lattice can always be established when the matrix lattice is subjected to a homogeneous deformation after regular but heterogeneous displacements of atomic planes. The parallel displacement of atomic planes takes place through the motion of the usual imperfect dislocations, while the homogeneous deformation is accompanied by the defects of other type. The homogeneous deformation is represented by a relation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\gamma} = \tau \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha}, \qquad (1)$$

where τ is a tensor correlating the coordinates of atoms in α-phase with those in γ-phase, and the positions of atoms in each phase are denoted by the coordinate axes fixed to definite atom rows of the phase in consideration.

There are, of course, various ways of deformation to convert the α-lattice to γ-lattice, but any one of these deformations can be produced from another by superposing a homogeneous deformation caused by the propagation of perfect or twin dislocations in every atomic plane. It is natural to suppose that the process will rather easily take place when the net amount of deformation is the smallest. Therefore, we shall define the standard transformation in (1) as that in which the number of pairs of the nearest neighbouring atoms in α-lattice still remaining the pairs of the nearest neighbouring atoms in γ-lattice is the maximum.

Since the energy of an interphase boundary depends markedly on its orientation as usually supposed, the interface will tend to become parallel to the plane of the lowest energy. During the progression of the transformation the low energy interface is difficult to migrate simultaneously in its normal direction on account of the large activation energy. The high energy barrier, however, may be avoided if the interface consists of steps of low energy plane and if each step migrates along the low energy interface. On the other hand, in a real crystal an interface usually reveals step structures owing to perfect dislocations being involved in the crystal. As easily supposed, the role of the step is very similar to that of the usual dislocation, and so we shall call these steps transformation dislocations.

* When an imperfect dislocation moves to the direction normal to its slip plane, neither generation nor annihilation of vacancy is accompanied, but some changes in interatomic distances as well as stacking faults are left behind the motion.

There are two types of transformation dislocations distinguished conveniently by positive and negative signs, that is, a pair of both kinds of transformation dislocations is annihilated when they meet in an interface, either of which cannot be formed independently of the other but always in a pair. Furthermore, usually a pair of transformation dislocations could not be created only by the released transformation energy, and so the dislocations will be created by means of a high stress concentration in such a way as the kinking[11], or be multiplied by a mechanism similar to Cottrell-Bilby's one for deformation twinning[12].

The motion of a transformation dislocation may be discussed in the same manner as Peierls-Nabarro's calculation[13]. Since the extension of the transformation dislocation is of the same order of magnitude as that of a perfect dislocation, the transformation dislocation moves in the closest atomic direction under a very small stress. Therefore, during the propagation of the transformation the low energy interface must be the plane of a rather large atomic density involving the closest atomic direction. This condition is satisfied in usual martensitic transformations, for example, the interface $(111)_F$ in the case of the transformation of a face-centred cubic lattice into a hexagonal close-packed structure.

The transformation associated with homogeneous deformation is performed by propagation of transformation dislocations in every atomic plane. The motion of dislocation of this type is very easy, because, when the transformation dislocation meets a perfect dislocation with a Burgers' vector crossing the low energy interface, it is multiplied in the successive atomic planes. Meanwhile, a perfect dislocation is not multiplied, and is subjected to the resistance against the motion by forming a jog in the above mentioned case. The transformation dislocation is thus subjected to less resistance due to the geometrical origin, but, owing to its large back stress field, it can move under very restricted circumstances. It may, therefore, be reasonable to suppose that the transformation dislocations move according to the mechanism of release of the strain energy introduced by their motion.

On the other hand, when the transformation process requires regular but heterogeneous displacements of atomic planes, as in the case of the transformation of a face-centred cubic structure into a hexagonal close-packed structure or of a body-centred cubic structure into a hexagonal close-packed structure, the usual imperfect dislocations should be propagated along a set of parallel atomic planes with a regular spacing. When these imperfect dislocations meet a perfect dislocation with a Burgers' vector crossing their sweeping planes, the imperfect dislocations are subjected to resistance by their jogs, because they usually climb a less distance than the spacing of atomic planes in which they should sweep.

(11) F. C. Frank and A. Stroh, Proc. Phys. Soc. London, **B 65** (1952), 811.
(12) A. H. Cottrell and B. A. Bilby, Phil. Mag., **42** (1951), 573.
(13) R. Peierls, Proc. Phys. Soc. London, **52** (1940), 34.
F. R. N. Nabarro, Proc. Phys. Phys. Soc. London, **59** (1947), 256.

The regular spacing of imperfect dislocations is then disturbed by the perfect dislocation, and stacking faults[14] will be left behind the transformation.

Some other points of difference between a transformation dislocation and a perfect dislocation are that the former should be propagated along the interface and that it cannot move repeatedly in a plane once swept, while the perfect dislocation is free from these restrictions.

In the transformation of a face-centred cubic structure into a body-centred tetragonal one, there are three ways to construct the relation (1), in which the pairs of the nearest neighbouring atoms in the body-centred tetragonal are always formed from the pairs of the nearest neighbours in the face-centred cubic structure. They are equivalent to one another, and so we shall use one of them which has been used by Frank[6], namely,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_M = \begin{pmatrix} 1 & 1 & 0 \\ \bar{1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A \tag{2}$$

The reciprocal relation is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_A = \frac{1}{2} \begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_M , \tag{3}$$

and the relations between planes are

$$\begin{pmatrix} h \\ k \\ l \end{pmatrix}_M = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ \bar{1} & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} h \\ k \\ l \end{pmatrix}_A \tag{4}$$

and

$$\begin{pmatrix} h \\ k \\ l \end{pmatrix}_A = \begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ k \\ l \end{pmatrix}_M , \tag{5}$$

where the subscripts A and M denote that the quantities correspond respectively to γ-phase, the face-centred cubic structure, and to α-phase, the body-centred tetragonal structure. According to these transformation relations, the Z-axis in γ-phase is transformed into the tetragonal axis in α-phase.

Since the transformation of a face-centred cubic structure into a body-centred tetragonal one is associated with an exclusively large strain, if the atom relations (2)~(4) are satisfied, there will be no way to avoid large strain energy even by taking a special shape of the product. The large strain energy can, however, be avoided if perfect dislocations move through the α-phase, and in this case the relations (2)~(4) do not hold, any atom relation in the form of (1) being no more correct. It is, however, convenient to use an approximate relation of the type of (1), even for the glide process, to know the strain of the martensite plates.

(14) O. S. Edwards and H. Lipson, J. Inst. Met., **69** (1943), 177; Proc. Roy. Soc., **180** (1942), 268. C. S. Barrett, *Plase Transformation in Solids*, (1951), 343.

After the motion of dislocations $a[uvw]$ on a set of planes parallel to (hkl) the final position of an atom is correlated to the initial position by the relation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\text{Def.}} = \begin{pmatrix} 1+hu & ku & lu \\ hv & 1+kv & lv \\ hw & kw & 1+lw \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\text{Init.}}, \qquad (6)$$

where the distance between the plane (hkl) and the origin of the coordinate is taken to be eqnal to the average distance of the planes, in which the dislocations proceed, and the Burgers' vector is defined as the vector signifying the displacement of the atoms in the plane (hkl) provided that the origin of the coordinate is fixed and that one of the dislocations moves in this plane.

III. Schiebung transformation

(a) Model

As already mentioned there are two different mechanisms in the transformation of a face-centred cubic structure into a body-centred tetragonal one and these two modes were called "Schiebung" and "Umklapp" transformations by Förster and Scheil[9]. Since the transformation dislocations can move easily through the material, provided that there is no back stress, it will be reasonable to suppose that the difference in the mechanism of transformation arises from the difference in the mechanism of releasing the back stress, against the motion of the transformation dislocations, by means of the motion of perfect dislocations.

If a perfect dislocation proceeds only within its slip plane, or even if it is multiplied in the plane, it cannot sufficiently release the back stress against the motion of the transformation dislocations, because the habit plane should be the slip plane, in which the plane strain is not released. Meanwhile, if perfect dislocations move in a set of planes parallel to the slip plane and at the same separation, α-phase will grow along a plane of low strain energy. There are two ways to create perfect dislocations in a set of planes with regular spacing, one being a static process, by which the length of the dislocation is increased continuously consuming the strain energy or directly the transformation energy, and the other being a dynamical process, by which the dislocation pairs are created consuming potential energy such as stacking fault energy or strain energy stored by freezing the kinetic energy of atoms during the transformation by means of a rather complicated process. According to Förster and Scheil, "Schiebung" transformation propagates more slowly than "Umklapp" transformation, and, therefore, the static process would correspond to the former transformation, while the dynamical process seems to correspond to the latter. In this section only the former process will be discussed.

A process increasing continuously the length of a dislocation is as follows: after the perfect dislocation has moved along the slip plane by a certain distance, its one part with screw character leaves the plane and arrives at the second plane separated by a definite distance from the first one, and then it forms a loop of dislocation on the second plane, and its screw part goes to the third plane, and

so on. In this way, many loops of dislocation may successively be formed in a set of planes.

Since the perfect dislocations are driven by stress field caused by transformation dislocations, the perfect dislocations will move through the body-centred cubic crystal. On the other hand, a perfect dislocation in a body-centred cubic structure can be split into two imperfect dislocations if it is in the plane $\{112\}_M$, that is,

$$\frac{a}{2}[111]_M \rightarrow \frac{a}{3}[111]_M + \frac{a}{6}[111]_M ,$$

and a layer of twinned structure may be produced between the two imperfect dislocations. The perfect dislocation, therefore, moves in $\{112\}_M$. According to the relation (4), $\{112\}_M$ can be formed from either $\{110\}_A$ or $\{113\}_A$, but the perfect dislocation moves in $\{112\}_M$ formed from $\{110\}_A$ on account of a better coherency between $\{112\}_M$ and $\{110\}_A$ than that between $\{112\}_M$ and $\{113\}_A$. When the relations (2)~(5) hold, four ways are possible for the motion of perfect dislocations, which, however, are similar to one another. In the following, therefore, will be discussed only one of them, in which the Burgers' vector of the perfect dislocation is $\frac{a}{2}[11\bar{1}]_M$, and the slip plane is $(112)_M$.

If there was no perfect dislocation except the $\frac{a}{2}[11\bar{1}]_M$ dislocation, the loops of perfect dislocation would take a suitable form to minimize the strain energy. There are, however, many perfect dislocations in the crystal forming networks with the density of about $10^8\,\mathrm{cm}^{-2}$, and the loops cannot expand beyond a certain distance in the direction perpendicular to the Burgers' vector but can extend freely in the vector direction, on account of the nature of jogs. The loop will thus become an oblonged form, the long dimension being along the Burgers' vector, and the habit plane should involve the direction of the Burgers' vector. α-phase will then grow conformably to the law of minimum strain energy, which may be expressed as dilatation-free along the habit plane in the perpendicular direction to the Burgers' vector and shear-free in the habit plane, because the dilatation in the direction of the Burgers' vector may be released by the motions of other dislocations.

(b) Transformation relations

On account of the motions of perfect dislocations the approximate atom relations are denoted by operating a tensor appeared in (6) with the relations (2)~(5). When a perfect dislocation $\frac{a}{2}[11\bar{1}]_M$ is propagated in a set of planes parallel to $(112)_M$ at each n-th atomic plane, the relation (6) becomes

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{Def.}} = \begin{pmatrix} 1+\frac{1}{2n} & \frac{1}{2n} & \frac{1}{n} \\ \frac{1}{2n} & 1+\frac{1}{2n} & \frac{1}{n} \\ -\frac{1}{2n} & -\frac{1}{2n} & 1-\frac{1}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{Init.}} . \qquad (7)$$

The approximate atom and plane relations between α-and γ-lattices in this model

will then be

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_M = \begin{pmatrix} 1 & 1+\frac{1}{n} & \frac{1}{n} \\ -1 & 1+\frac{1}{n} & \frac{1}{n} \\ 0 & -\frac{1}{n} & 1-\frac{1}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A , \tag{8}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_A = \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 \\ 1-\frac{1}{n} & 1-\frac{1}{n} & -\frac{2}{n} \\ \frac{1}{n} & \frac{1}{n} & 2+\frac{2}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_M , \tag{9}$$

$$\begin{pmatrix} h \\ k \\ l \end{pmatrix}_M = \frac{1}{2} \begin{pmatrix} 1 & 1-\frac{1}{n} & \frac{1}{n} \\ -1 & 1-\frac{1}{n} & \frac{1}{n} \\ 0 & -\frac{2}{n} & 2+\frac{2}{n} \end{pmatrix} \begin{pmatrix} h \\ k \\ l \end{pmatrix}_A , \tag{10}$$

$$\begin{pmatrix} h \\ k \\ l \end{pmatrix}_A = \begin{pmatrix} 1 & -1 & 0 \\ 1+\frac{1}{n} & 1+\frac{1}{n} & -\frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & 1-\frac{1}{n} \end{pmatrix} \begin{pmatrix} h \\ k \\ l \end{pmatrix}_M . \tag{11}$$

(c) Habit plane

The habit plane should involve the Burgers' vector of the perfect dislocation $\frac{a}{2}[11\bar{1}]_M$, i.e. $\frac{a}{2}[0\bar{1}1]_A$, and then the plane is denoted by $(1XX)_A$, where X is a variable to be determined from the requirement of the minimum strain energy. The line perpendicular to $[0\bar{1}1]_A$ in $(1XX)_A$ is $[\overline{2X}\,1\,1]_A$, which is converted into $\left[1+\frac{2}{n}-2X,\ 1+\frac{2}{n}+2X,\ 1-\frac{2}{n}\right]_M$ after the transformation. The converted direction should be perpendicular to $[11\bar{1}]_M$, and then we shall have

$$\frac{2}{n}(2a_M^2+c_M^2)+2a_M^2-c_M^2=0 ,$$

or

$$\frac{2}{n}=\frac{-2+\alpha^2}{2+\alpha^2} , \tag{12}$$

where

$$\alpha=\frac{c_M}{a_M} .$$

Here a_M and c_M are the lattice constants of α-phase respectively in x- and z-directions.

The length of the lattice vector $[\overline{2X}\,1\,1]$ is

$$l_A=\sqrt{2}\sqrt{1+2X^2}\ a_A ,$$

and after the transformation, it becomes,

$$l_M = \frac{2\sqrt{2}}{2+\alpha^2}\sqrt{\alpha^4+(2+\alpha^2)^2X^2+2\alpha^2}\ \ a_M\ .$$

These two lengths l_A and l_M are equal to each other when the α-phase is dilatation-free in the direction $[\overline{2X}\,1\,1]_A$, and we can obtain

$$X = \pm\sqrt{\frac{2+\alpha^2-4\alpha^2\eta^2}{(2+\alpha^2)(4\eta^2-2)}} \tag{13}$$

where

$$\eta = \frac{a_M}{a_A}$$

Inserting suitable values into (12) and (13), we can obtain numerical results to be compared with observations. Table 1 and 2 show the results respectively for Fe-Ni alloys and for plain carbon steels. In Fe-C alloys, the difference between the calculated and observed habit planes is greater than the error in the experiments[8]. The deviations, however, may be neglected in comparison with the simplified assumption concerning the requirement of the minimum strain energy.

Table 1. Habit Plane in Schiebung Transformation (Fe-Ni alloys)

Ni in wt%	a_M in Å	a_A in Å	X	n	Habit Plane
20	2.8688	3.589	±0.516	−6	$\{3.9, 2, 2\}_A$
25	2.8655	3.581	±0.511	−6	$\{3.9, 2, 2\}_A$
30	2.8632	3.576	±0.507	−6	$\{3.9, 2, 2\}_A$

Table 2. Habit Plane in Schiebung Transformation (Fe-C alloys)

C in wt%	a_M in Å	c_M in Å	a_A in Å	X	n	Habit Plane
0.8	2.846	2.954	3.584	±0.473	−6.667	$\{4.2, 2, 2\}_A$
1.0	2.846	2.979	3.593	±0.467	−6.850	$\{4.3, 2, 2\}_A$
1.2	2.846	3.004	3.601	±0.460	−7.040	$\{4.3, 2, 2\}_A$
1.4	2.846	3.028	3.610	±0.456	−7.217	$\{4.4, 2, 2\}_A$

(d) Lattice relation

Since the habit plane $(1XX)_A$ turns into $\frac{1}{2}(1+X, -1+X, 2X)_M$ by the relation (10), we have

$$(1XX)_A \parallel (1+X,\ -1+X,\ 2X)_M, \tag{14}$$

and, as already mentioned,

$$[0\bar{1}1]_A \parallel [11\bar{1}]_M \tag{15}$$

The two relations (14) and (15) are sufficient to give the lattice relation between both phases. Tables 3 and 4 show the angle between a close-packed atomic plane and the habit plane in the cases respectively of Fe-Ni and Fe-C alloys. When X is positive, $(111)_A$ is parallel to $(101)_M$ within 1/2°, and when X is negative,

Table 3. Lattice Relation in Schiebung Transformation (Fe-Ni alloys)

Ni in wt%	X	$\angle(1XX)_A(111)_A$ or $\angle(1XX)_A(\bar{1}11)_A$	$\angle(1+X,-1+X,2+X)_M(101)_M$ or $\angle(1+X,-1+X,2+X)_M(011)_M$	$\angle(111)_A(101)_M$ or $\angle(\bar{1}11)_A(011)_M$
20	±0.516	18°37′	18°12′	25′
25	±0.511	18°53′	18°29′	24′
30	±0.507	17°05′	18°42′	23′

Table 4. Lattice Relation in Schiebung Transformation (Fe-C alloys)

C in wt%	X	$\angle(1XX)_A(111)_A$ or $\angle(1XX)_A(\bar{1}11)_A$	$\angle(1+X,-1+X,2+X)_M(101)_M$ or $\angle(1+X,-1+X,2+X)_M(011)_M$	$\angle(111)_A(10\bar{1})_M$ or $\angle(\bar{1}11)_A(011)_M$
0.8	±0.473	20°57′	20°35′	22′
1.0	±0.467	21°17′	20°55′	22′
1.2	±0.461	21°38′	21°16′	22′
1.4	±0.456	21°55′	21°33′	22′

$(\bar{1}11)_A$ is parallel to $(011)_M$ within 1/2°. Therefore, α- and γ- crystals satisfy Kurdjumow-Sachs' relation[15] in this case.

(e) Deformation associated with transformation

We shall next calculate only the shear along the habit plane, without considering other components of deformation. The shear is obtained by calculating the final direction of an atom row, which was originally perpendicular to the habit plane. The atom row perpendicular to the habit plane $[1XX]_A$ turns into $[1+X+\frac{2}{n}X,\ -1+X+\frac{2}{n}X,\ (1-\frac{2}{n})X]_M$ by (8) after the transformation. The angle between this direction and $[1+X,\ -1+X,\ 2X]_M$ gives the amount of deformation, and the intersection of the habit plane and the plane passing through these two directions gives the direction of shear. A numerical result of calculation in the case of 1.4 wt. per cent carbon steel was as follows: the shear direction was $[\overline{0.831}\ 0.695\ 1.12\bar{9}]_A$ for the habit plane $(4.37\ 2\ 2)_A$, the amount of shear being about 0.187. The results seems to agree with Bowles' observation, though he did not give explicitly the shear direction and the amount of shear[3].

IV. Umklapp transformation

(a) Model

According to the model of "Schiebung" transformation, the perfect dislocation becomes difficult to move, as the transformation proceeds, due to the increase in jog. In order to continue the transformation, another dislocation having the same Burgers' vector as the former should move continuing the release of the strain energy. Consequently, in the course of the transformation there must be a stress field sufficient to make a perfect dislocation move. The force required to move a perfect dislocation, however, increases rapidly with the depression of temperature and, accordingly, the stress may be increased sufficiently to cause deformation twinning. Since the motion of a perfect dislocation is far slower than the sound velocity, while the deformation twin propagates very rapidly, perhaps with a fraction of the sound velocity, another mechanism with far greater propagation rate than that in the mechanism proposed in Sec. III may be possible.

From a simple calculation it can easily be shown that the transformation is impossible on account of large strain energy when the interface between α- and γ-phases is parallel to a plane of low indices such as a slip plane or a twinning plane. Besides, as mentioned in Sec. III, the strain energy could not sufficiently

(15) G. Kurdjumow and G. Sachs, Z. Phys., **64** (1930), 325.

be released only by means of irrational habit planes, if the perfect dislocations were not produced in a set of parallel planes. Therefore, in the mechanism of rapid propagation numerous dislocation pairs should cataclysmatically be created in a set of parallel planes with a regular spacing.

The dynamical process simililar to twinning should be associated with motions of imperfect dislocations in every atomic plane resulting in a homogeneous deformation. The reason for this will be seen from Sec. II. If the imperfect dislocations were replaced by the perfect dislocations or by those with greater strength than these, the velocity of propagation would be far smaller than the sound velocity, because the perfect dislocations can move in the same atomic plane repeatedly and the homogeneous distribution of dislocations would be disturbed as the result of occasional multiplication in the same plane by interacting with dislocations belonging to other slip systems.

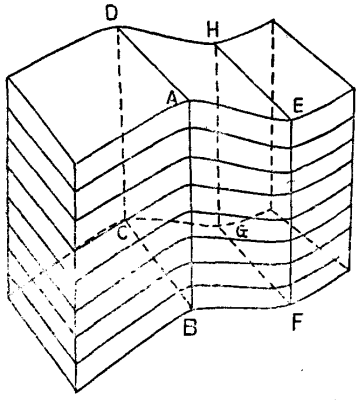

Fig. 1. Formation of high energy stacking.

A plausible mechanism satisfying the above-mentioned requirements may be as follows: let us suppose that in a body-centred crystal the region between the planes ABCD and EFGH in Fig. 1 is subjected to homogeneous displacement of atomic planes and is left in the state of a high energy stacking, which can be caused by consuming the kinetic energy. The high energy stacking is not stable and tends to take a stable

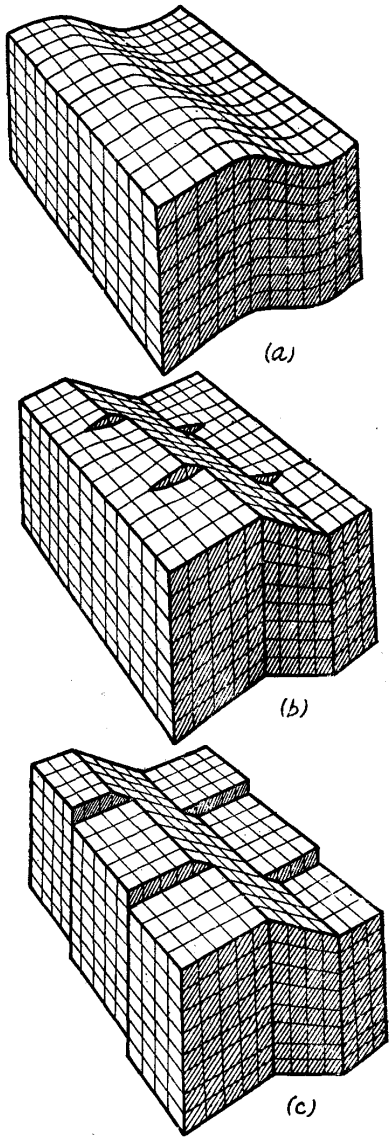

Fig. 2. Schematic representation of the successive stages of formation of an array of perfect dislocations.
(a) First stage of homogeneous deformation, atomic planes in the deformed layer are in high energy stacking.
(b) Formation of perfect dislocations taking a stable stacking in the homogeneously deformed layer.
(c) Propagation of perfect dislocations.
In these figures screw dislocations are produced, but as easily seen, if the boundary between the homogeneously deformed layer and its surrounding rotates round the horizontal line of the figure, the created dislocations become the mixed dislocations.

configuration with the normal stacking producing arrays of perfect dislocations along the plane ABCD or EFGH. Since they are not usually the planes of the lowest interface energy between the homogeneously deformed and undeformed regions in the body-centred crystal, perfect dislocations produced in the plane ABCD or EFGH are driven to leave the plane and to proceed through the undeformed region of body-centred tetragonal lattice. The situations can be seen more clearly in Fig. 2.

The high energy stacking is produced by consuming the kinetic energy supplied by suitable mechanisms as already mentioned. The initial source of the kinetic energy may be supplied by the usual deformation twinning, which takes place in a small body-centred crystal produced by means of "Schiebung" transformation. The continuous supply of kinetic energy is possible only through the transformation dislocations, that is, if a pair of transformation dislocations approaches each other along an interphase boundary, they cannot stop their motion at the time when they meet, but continue to move accross each other by a certain distance leaving behind a stacking fault. This process will naturally take place if, during the growth of α-phase, the both sides of the high energy stacking layer proceed farther than the layer as shown in Fig. 3.

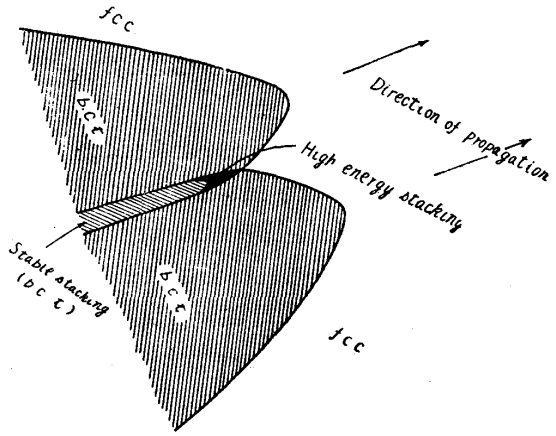

Fig. 3. Schematic representation of the end of a martensite plate in Umklapp transformation.

Let us suppose that the transformation dislocations proceed in the plane $(1\bar{1}2)_M$ for the reason mentioned in the previous section, then atomic layers $(1\bar{1}2)_M$ stack as shown in the third column of Table 5, in which the stackings are represented

Table 5. Changes in Stacking

F. C. C. $(011)_A$	B. C. T. $(1\bar{1}2)_M$	High energy stacking	Stable stacking (macroscopic strain is not relieved)	Stable stacking (macroscopic strain is removed perfectly)
0	0	0	0	0
$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$	$-\frac{1}{6}$	$-\frac{1}{6}+\frac{1}{2}$
$\frac{2}{4}$	$\frac{2}{6}$	$\frac{2}{12}$	$-\frac{2}{6}+\frac{1}{2}$	$-\frac{2}{6}+\frac{1}{2}$
$\frac{3}{4}$	$\frac{3}{6}$	$\frac{3}{12}$	$-\frac{3}{6}+\frac{1}{2}$	$-\frac{3}{6}+\frac{2}{2}$
$\frac{4}{4}$	$\frac{4}{6}$	$\frac{4}{12}$	$-\frac{4}{6}+\frac{2}{2}$	$-\frac{4}{6}+\frac{3}{2}$
$\frac{5}{4}$	$\frac{5}{6}$	$\frac{5}{12}$	$-\frac{5}{6}+\frac{2}{2}$	$-\frac{5}{6}+\frac{3}{2}$
$\frac{6}{4}$	$\frac{6}{6}$	$\frac{6}{12}$	$-\frac{6}{6}+\frac{3}{2}$	$-\frac{6}{6}+\frac{4}{2}$

by taking the modulus as unit. In the stable stacking, $(1\bar{1}2)_M$ planes would repeat the same stacking at every sixth time and a body-centred tetragonal lattice would again be established. If the macroscopic strain in the high energy stacking layer is not relieved during the process of reaching the stable configuration, the stacking in the fourth column is then established. Meanwhile, if the strain is perfectly removed, the stacking will be represented in the final column. Those stable stackings in the fourth and fifth columns are attained from the high energy stacking through twinning or slip.

The interface energy between the undeformed region and the high energy stacking layer in the body-centred tetragonal crystal will become minimum when the configuration of atoms is symmetric concerning the interface, that is, the interface reaches the minimum energy state when it becomes a twin boundary in the body-centred tetragonal structure involving an array of perfect or imperfect dislocations arranged vertically to their Burgers' vector. As seen from Table 5, if the macroscopic strain is not relieved, the perfect dislocations will be at every two planes, and the lowest energy interface will be $(332)_M$, while if it is perfectly relieved, the lowest energy interface will be $(111)_M$. Actually the planes of the lowest interface energy may be between these two.

An array of perfect dislocations, which is to move through the body-centred structure, are created along the intersections of their slip planes and the low energy interface. The intersections, therefore, should nearly be a dilatation-free direction during the transformation. The number of the perfect dislocations, which depends on the angle between the low energy interface and the interface of the high energy stacking layer, should nearly be that required just to remove the strain energy of the martensite plate. The restriction concerning the number of dislocations will be considered toward the end of this section. The dilatation along the pefrect dislocation line during the transformation can easily be estimated by comparing the lengths between the lattice vector in b. c. t. along the dislocation line and its converted lattice vector in f. c. c. by (3). Then, it can be shown that the most suitable slip plane for the above-mentioned requirement are $(\bar{1}12)_M$ or $(1\bar{1}2)_M$.

In the following, four cases will be discussed, namely, $(332)_M$ the low energy interface and $(\bar{1}12)_M$ slip plane, $(111)_M$ $(\bar{1}12)_M$, $(332)_M$ $(1\bar{1}2)_M$, and finally $(111)_M$ $(1\bar{1}2)_M$.

(b) Transformation relations

The transformation relations showing the atom relations in some averaged scale can be given in the same way as in the case of "Schiebung" transformation.

(i) and (ii) $(\bar{1}12)_M$ slip. In these cases the Burgers' vector is $\frac{a}{2}[1\bar{1}1]_M$, and then we shall have

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\text{Def.}} = \begin{pmatrix} 1-\frac{1}{2n} & \frac{1}{2n} & \frac{1}{n} \\ \frac{1}{2n} & 1-\frac{1}{2n} & -\frac{1}{n} \\ -\frac{1}{2n} & \frac{1}{2n} & 1+\frac{1}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\text{Init.}} . \tag{16}$$

Therefore,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_M = \begin{pmatrix} 1-\frac{1}{n} & 1 & \frac{1}{n} \\ -1+\frac{1}{n} & 1 & -\frac{1}{n} \\ -\frac{1}{n} & 0 & 1+\frac{1}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A , \tag{17}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_A = \frac{1}{2} \begin{pmatrix} 1+\frac{1}{n} & -1-\frac{1}{n} & -\frac{2}{n} \\ 1 & 1 & 0 \\ \frac{1}{n} & -\frac{1}{n} & 2-\frac{2}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_M , \tag{18}$$

$$\begin{pmatrix} h \\ k \\ l \end{pmatrix}_M = \frac{1}{2} \begin{pmatrix} 1+\frac{1}{n} & 1 & \frac{1}{n} \\ -1-\frac{1}{n} & 1 & -\frac{1}{n} \\ -\frac{2}{n} & 0 & 2-\frac{2}{n} \end{pmatrix} \begin{pmatrix} h \\ k \\ l \end{pmatrix}_A , \tag{19}$$

$$\begin{pmatrix} h \\ k \\ l \end{pmatrix}_A = \begin{pmatrix} 1-\frac{1}{n} & -1+\frac{1}{n} & -\frac{1}{n} \\ 1 & 1 & 0 \\ \frac{1}{n} & -\frac{1}{n} & 1+\frac{1}{n} \end{pmatrix} \begin{pmatrix} h \\ k \\ l \end{pmatrix}_M . \tag{20}$$

(iii) and (iv) $(1\bar{1}2)_M$ slip. In these cases the Burgers' vector is $\frac{a}{2}[\bar{1}11]_M$, and then we shall have

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\text{Def.}} = \begin{pmatrix} 1-\frac{1}{2n} & \frac{1}{2n} & -\frac{1}{n} \\ \frac{1}{2n} & 1-\frac{1}{2n} & \frac{1}{n} \\ \frac{1}{2n} & -\frac{1}{2n} & 1+\frac{1}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\text{Init.}} . \tag{21}$$

Therefore,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_M = \begin{pmatrix} 1-\frac{1}{n} & 1 & -\frac{1}{n} \\ -1+\frac{1}{n} & 1 & \frac{1}{n} \\ \frac{1}{n} & 0 & 1+\frac{1}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A , \tag{22}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_A = \frac{1}{2} \begin{pmatrix} 1+\frac{1}{n} & -1-\frac{1}{n} & \frac{2}{n} \\ 1 & 1 & 0 \\ -\frac{1}{n} & \frac{1}{n} & 2-\frac{2}{n} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_M , \tag{23}$$

$$\begin{pmatrix} h \\ k \\ l \end{pmatrix}_M = \frac{1}{2}\begin{pmatrix} 1+\frac{1}{n} & 1 & -\frac{1}{n} \\ -1-\frac{1}{n} & 1 & \frac{1}{n} \\ \frac{2}{n} & 0 & 2-\frac{2}{n} \end{pmatrix}\begin{pmatrix} h \\ k \\ l \end{pmatrix}_A , \tag{24}$$

$$\begin{pmatrix} h \\ k \\ l \end{pmatrix}_A = \begin{pmatrix} 1-\frac{1}{n} & -1+\frac{1}{n} & \frac{1}{n} \\ 1 & 1 & 0 \\ -\frac{1}{n} & \frac{1}{n} & 1+\frac{1}{n} \end{pmatrix}\begin{pmatrix} h \\ k \\ l \end{pmatrix}_M . \tag{25}$$

(c) Habit plane

(i) $(332)_M$ low energy interface, $(\bar{1}12)_M$ slip plane

The intersection between the slip plane and the low energy interface is $[2\bar{4}3]_M$ which corresponds to $[3\bar{1}3]_A$ by (18). The direction perpendicular to this line in the face-centred cubic lattice is $[X, 3(1+X), 1]_A$, which turns into $[3+4X+\frac{1}{n}(1-X), 3+2X-\frac{1}{n}(1-X), 1+\frac{1}{n}(1-X)]_M$ by (17), where X is a variable. If the line $[X, 3(1+X), 1]_A$ is in the habit plane, we shall have

$$\frac{1}{n}(1-X) = \frac{2-\alpha^2}{2+\alpha^2}$$

due to the requirement of shear-free in the martensite plane, where

$$\alpha = \frac{c_M}{a_M}$$

Since the martensite plate is dilatation-free in the direction $[X, 3(1+X), 1]_A$, we can obtain the plane in the same way as in the case of the "Schiebung" transformation, that is,

$$X = B \pm \sqrt{B^2 - 1} ,$$

where

$$B = \frac{-8(5+2\alpha^2)\eta^2 + 9(2+\alpha^2)}{2(2+\alpha^2)(10\eta^2-5)} , \tag{27, i}$$

and

$$\eta = \frac{a_M}{a_A} ,$$

and, accordingly, the habit plane will be $(10+9X, 3-3X, \overline{9+10X})_A$.

(ii) $(111)_M$ low energy interface, $(\bar{1}12)_M$ slip plane

The two relations determining n and X are

$$\frac{1}{n}(1-X) = \frac{2-\alpha^2}{2+\alpha^2} ,$$

$$X = B \pm \sqrt{B^2-1} , \quad B = \frac{-2(10+3\alpha^2)\eta^2 + 4(2+\alpha^2)}{(2+\alpha^2)(10\eta^2-5)} . \tag{27, ii}$$

The habit plane is $(5+4X, 2-2X, \overline{4+5X})_A$.

(iii) $(332)_M$ low energy interface, $(1\bar{1}2)_M$ slip plane

$$\frac{1}{n}(1+X) = \frac{2-\alpha^2}{2+\alpha^2}$$

$$X = B \pm \sqrt{B^2-1}\,,\quad B = \frac{8(5+2\alpha^2)\eta^2-9(2+\alpha^2)}{2(2+\alpha^2)(10\eta^2-5)}\,. \qquad (27,\ \text{iii})$$

The habit plane is $(10-9X,\ -3-3X,\ 9-10X)_A$.

(iv) $(111)_M$ low energy interface, $(1\bar{1}2)_M$ slip plane

$$\frac{1}{n}(1+X) = \frac{2-\alpha^2}{2+\alpha^2}$$

$$X = B \pm \sqrt{B^2-1}\,,\quad B = \frac{2(10+3\alpha^2)\eta^2-4(2+\alpha^2)}{(2+\alpha^2)(10\eta^2-5)} \qquad (27,\ \text{iv})$$

The habit plane is $(5-4X,\ -2-2X,\ 4-5X)_A$.

By inserting appropriate values of η and α in these expressions of the habit plane, we shall compare the calculations with the observations. In the Fe-Ni alloy (30 wt per cent Ni) $\alpha = 1$, $\eta = 0.8007$ and in the Fe-C alloy (1.7 wt per cent C) $\alpha = 1.0774$, $\eta = 0.7858$, from which we obtained the values of X and n shown in Table 6 for the above-mentioned four cases. The habit planes corresponding to these values of X are also shown in the table.

Table 6. Values of X and n in Umklapp Transformation

		(i)		(iii)	
Fe-Ni (30 wt% Ni)	X	−1.376	−0.727	0.727	1.376
	n	7.13	5.18	5.18	7.13
Fe-C (1.7 wt% C)	X	−1.342	−0.745	0.745	1.342
	n	8.82	6.57	6.57	8.82
		(ii)		(iv)	
Fe-Ni (30 wt% Ni)	X	−1.568	−0.638	0.638	1.568
	n	7.70	4.91	4.91	7.70
Fe-C (1.7 wt% C)	X	−1.484	−0.674	0.674	1.484
	n	9.36	6.30	6.30	9.36

There are two different values of X satisfying each of the expressions (27, i)∼(27, iv). The smaller one in absolute magnitude, however, cannot be realized, because, if it were the case, the angle between the habit plane and the low energy interface would not be suitable for the production of the required number of perfect

Table 7. Habit Plane in Umklapp Transformation

	Fe-Ni alloy (30 wt% Ni)	Fe-C alloy (1.7 wt% C)
(i) and (iii)	$(\bar{2}.00,\ 5.98,\ 4.00)_A$, $(4.00,\ 5.98,\ \bar{2}.00)_A$	$(\bar{2}.00,\ 6.76,\ 4.25)_A$, $(4.25,\ 6.76,\ \bar{2}.00)_A$
(ii) and (iv)	$(\bar{2}.00,\ 8.06,\ 6.04)_A$, $(6.04,\ 8.06,\ \bar{2}.00)_A$	$(\bar{2}.00,\ 10.60,\ 7.30)_A$, $(7.30,\ 10.60,\ \bar{2}.00)_A$

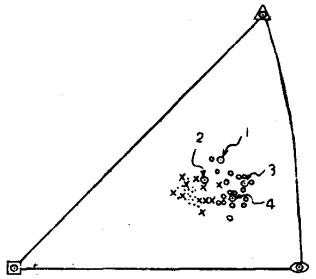

Fig. 4. Comparison between the calculated habit planes and the observations by Grenninger and Troiano.
• Fe-C
× Fe-Ni-C
∘ Fe-Ni
} Observed habit planes by Greninger & Troiano
⊙ Calculated habit plane
1 (2.00, 4.25, 6.76)
2 (2.00, 7.30, 10.60)
} Fe-C alloy
3 (2.00, 4.00, 5.98)
4 (2.00, 6.04, 8.06)
} Fe-Ni alloy

dislocations, as easily shown by straightforward calculations.

The calculated habit planes in the table are close to the observed plane $(\hat{2}59)_A$. Fig. 4 shows the observed habit planes by Greninger and Troiano[16] and the results of the present calculation.

(d) Lattice relation

(i) $(332)_M$ low energy interface, $(\bar{1}12)_M$ slip plane

At the habit plane,

$$(10+9X,\ 3-3X,\ \overline{9+10X})_A \parallel \left(\frac{14+6\alpha^2}{2+\alpha^2}+3X,\ \frac{-8-3\alpha^2}{2+\alpha^2}-6X,\ \frac{-20-8\alpha^2}{2+\alpha^2}-10X\right)_M$$

In the Fe-Ni alloy (30 wt. per cent Ni), $\alpha = 1$, and the habit plane is approximated by $(1\bar{3}\bar{2})_A$, and, accordingly,

$$(1\bar{3}\bar{2})_A \parallel (2{,}140\ 3{,}860\ 3{,}720)_M$$

The direction of the perfect dislocation lines is invariant through the transformation, that is,

$$[3\bar{1}3]_A \parallel [2\bar{4}3]_M$$

The angle between $(1\bar{3}\bar{2})_A$ and $(\bar{1}11)_A$ is 22°13′ and the angle between (2.140, 3.860, 3.720)$_M$ and $(011)_M$ is 21°47′. Since the two planes $(\bar{1}11)_A$ and $(011)_M$ meet each other nearly in the habit plane, the angle between $(\bar{1}11)_A$ and $(011)_M$ is only about 26′, while the angle between $[1\bar{1}2]_A$ and $[01\bar{1}]_M$ is 2°39′.

(ii) $(111)_M$ low energy interface, $(\bar{1}12)_M$ slip plane

The angle between $(\bar{1}11)_A$ and $(011)_M$ is about 37′ and the angle between $[1\bar{1}2]_A$ and $[01\bar{1}]_M$ is about 2°5′.

These results seem to show that the lattice relation becomes Greninger-Troiano's one[16] in "Umklapp" transformation.

(e) Deformation associated with transformation

Since in the Fe-Ni alloy the calculated habit plane shows a better agreement with the observation in the case of $(111)_M$ the low energy interface, than in the case of $(332)_M$, the deformation associated with the transformation will be calculated only for the former case. The calculated habit plane is then $(\bar{1}\ 4.03\ 3.\ 02)_A \rightleftarrows (3.\ 92\ 4.77\ 5.52)_M$. The atom row $[\bar{1}\ 4.03\ 3.\ 02]_A$ perpendicular to the habit plane in the face-centred cubic structure turns into $[3.55\ 4.51\ 3.54]_M$ through the transformation. The shear direction is then $[2\ 5.1\ \overline{6.2}]_A$, and the amount of shear is 0.217.

The result shows a good agreement with Machlin-Cohen's observation[4]; according to their result, the shear direction is $[\bar{1}\bar{5}6]_A$ for the habit plane of $(259)_A$. By the permutation $(259)_A \rightarrow (\bar{2}95)_A$, $[\bar{1}\bar{5}6]_A$ turns into $[16\bar{5}]_A$. The difference between $[2\ 5.1\ \overline{6.2}]_A$ and $[16\bar{5}]_A$ is the order of error arising from the difference between the habit planes in Machlin-Cohen's and in the present case.

V. Discussions

(a) Motions of imperfect or transformation dislocations

The concept "propagation of shear wave" accepted by Bowles[3] and Machlin-

(16) A. B. Greninger and A. R. Troiano, Trans. AIME., **145** (1941), 289; **185** (1949), 590.

Cohen[4] should rigorously be replaced by the concept of propagation of transformation dislocations, for the reason that the shear wave cannot be extended so widely as the atomic structure of the wave being negligible. The inadequacy of such a phenomenological concept as shear wave is very accentuated in the transformation of the face-centred bubic structure into the hexagonal close-packed structure[5]. In this case the transformation proceeds when half dislocations are situated at every two octahedral planes and move in these planes.

The concepts "first shear" and "second shear" are only for the convenience of the representation of experiment. It is, therefore, meaningless to suppose that the first shear propagates before the second shear takes place. As already mentioned, the perfect dislocations move to release the stress caused by the motion of the transformation dislocations. Therefore, the intermediate structure produced by the first shear, which was supposed to be homogeneous even in atomic scale, has no physical significance.

(b) $\{225\}_A$ and $\{259\}_A$ habit planes

A martensite plate is produced on $\{225\}_A$ or $\{259\}_A$, depending upon the initial direction and length of the perfect dislocation which moves to release the transformation stress. When the length is very small, the habit plane involve the Burgers' vector of the perfect dislocation, while in the case of a long dislocation, the habit plane should include the initial direction of the dislocation. The production of long jog-free dislocations, being not the pure screw, however, requires a very complicated mechanism as mentioned in the previous section. Therefore, the difference in the habit plane can be explained only on the basis of the essential difference in propagation mechanism. Indeed, the process of transformation reveals an extraordinary difference between both cases having $(225)_A$ and $(259)_A$ habit planes. Attempts to explain the difference in the habit plane on the basis of the difference in some properties such as the interface energy would fail.

(c) Creation of perfect dislocations in the case of $\{225\}_A$ habit plane.

Frank suggested that the perfect dislocations necessary for the transformation would be produced in a way which resembles the mechanism of kinking[6]. In the case of $\{225\}_A$ habit plane, however, the perfect dislocations are of screw type, and could not form a stable boundary of kink band under such a large stress field as to enable it to create a dislocation pair at its end. The reason for this is that a screw dislocation is not restricted within one plane as $(\bar{1}12)_M$, but tends to move along the martensite plate releasing the stress field about it, and there is no stress sufficient to create the pair at the end of the martensite plate.

(d) Note on "Burst"

As Machlin-Cohen[18] have shown, in a single crystal of Fe-Ni alloy with 31 wt. per cent Ni, the transformation takes place cataclysmically over the whole sample with an audible click. The transformation similar to this also takes place in a coarse grained material showing $(259)_A$ habit plane. According to the present

(17) E. S. Machlin and M. Cohen; J. Metals, (1951), 746.

author's model, the martensite plate with the habit $(259)_A$ is formed under the supply of kinetic energy, and seems to propagate very rapidly at a rate comparable to the sound velocity of the material. Therefore, once a plate has been formed, the shock wave will induce the growth of martensite plates. If the shock wave from a martensite plate could induce the growth of one or more plates, the transformation should be observed as the "Burst". Meanwhile, it is well known that the larger the crystal, the larger the martensite plate is. In a coarse-grained sample, therefore, a martensite plate can emit large amounts of kinetic energy sufficient to produce the "Burst", while if the grain size is smaller than a certain critical value, martensite plates will simultaneously be formed only in a region neighbouring the plate formed in an initial stage.

On the other hand, in the transformation resulting in the habit plane $\{225\}_A$, the dislocation goes to the next slip plane after the formation of a dislocation loop. The dislocation, therefore, should lose its kinetic energy when it removes from one slip plane to another, and the "Schiebung" transformation showing the habit plane $\{225\}_A$ propagates quasi-statically.

It is clear that the criterion whether the process is dynamic or static cannot be given only by the number of embryos producing the same direction of atom movement as Machlin-Cohen suggested[17], because their explanation should assume a critical value for the amount of kinetic energy emitted from the martensite plate.

(e) Microstructure of martensite with habit $\{259\}_A$

The perfect dislocations, which are produced when the high energy stacking layer is converted into the stable one, move releasing the strain energy and increasing the width of the martensite plate. The perfect dislocations, however, become difficult to move on account of the increase in the number of jogs as in the case of "Schiebung" transformation. The martensite plate, therefore, will take a lense shape if it is smaller than a certain magnitude, and the width will become a definite value independent of the size of the plate, if it is larger than this.

The midrib, the characteristic of the martensite plate, is the region of a different crystal orientation than the martensite itself, left after the transition of the high energy stacking. It is easily seen that, if the martensite plate takes a lense shape, the lattice relation in Sec. IV (d) does not hold. Then, it may be possible for lattice relation to become Nishiyama's one[18], but the present theory cannot discuss such possibility.

(f) A note on "Schiebung" transformation in a free surface

According to the observation on the direction of martensite plates in the free surface in the case of "Schiebung" transformation in Fe-Ni alloys, these directions are not the intersections of a definite family of habit planes with the free surface[10]. Since the martensite plate grows in the direction corresponding to the minimum strain energy, the situation of the affair will markedly differ between the interior

(18) Z. Nishiyama, Sci. Rep. Tôhoku Univ., **23** (1934), 638.

and the free surfaces of the crystal. Bowles, however, reported the habit $\{225\}_A$ in free surface in 1.35 wt. per cent carbon steel[3]. The Cottrell's atmosphere of carbon atoms around the perfect dislocations may perhaps make the situation in carbon steels different from that in Fe-Ni alloys. This problem requires further investigations.

(g) The occurrence of transformation

Based on the nucleation and growth theory[19] or on the strain embryo[20], many discussions have been accumulated concerning the condition for the occurrence of the transformation. These treatments, however, are too phenomenological and these concepts should be discussed from atomistic point of view.

As already mentioned, the transformation takes place according to the conditions of formation and motion of perfect dislocations and, especially, the interaction between solute atoms and perfect dislocations seems to result in the different mechanisms of transformation, such as "Schiebung" and "Umklapp". These interactions of solute atoms may play the most important role in the criterion of initiation of the transformation. The investigation on this subject, however, is now in progress.

In conclusion the auther wishes to thank to Prof. Sakae Takeuchi for his interest and support and also to Mr. Toshio Homma for his valuable discussions on the recent experimental studies on martensitic transformation.

(19) J. C. Fisher, J. H. Hollomon and D. Turnbull, Trans. AIME., **185** (1949), 691.

(20) M. Cohen, E. S. Machlin and V. G. Paranjpe, *Thermodynamics in Physical Metallurgy* (ASM, 1950), 242.